### Dark energy domination in the local flow of giant galaxies

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A dozen most luminous galaxies at distances up to 10 Mpc from the Local Group are moving away from the group forming the local expansion flow of giants. We use recent Hubble Space Telescope data on the local giants and their numerous fainter companions to study the dynamical structure and evolutionary trends of the flow. A N-body computer model that reproduces the observed kinematic of the flow is constructed under the assumption that the flow is embedded in the universal dark energy background. In the model, the motions of the flow members are controlled by their mutual attraction force and the repulsion force produced by the dark energy. It is found that the dark energy repulsion dominates the force field of the flow. Because of this, the flow expands with acceleration quantified with the local deceleration parameter. The dark energy domination is enhanced by the environment effect of the low mean matter density on the spatial scale of 50 Mpc in the local universe. The accelerating expansion cools the flow and introduces to it a nearly linear velocity-distance relation with the time-rate that is close to Hubble's factor of the global cosmological expansion.

Keywords: Galaxies, groups and clusters of galaxies; local flows of galaxies; dark matter and dark energy

# 1 Introduction

Expansion flows of galaxies were discovered and studied on spatial scales from  $\simeq 1$  to  $\simeq 300$  Mpc (Sandage 1986; Sandage et al. 1972, 1999, 2006). The kinematic of

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the flows follows rather closely the linear velocity-distance relation  $V \simeq HR$  known as Hubble's law. They are considered cold in the sense that the deviations from this relation are relatively small and the corresponding radial velocity dispersion  $\sigma$  is low in the observed flows.

The nearest and best studied (Sandage et al. 1972; Karachentsev et al. 2002, 2003a, 2009; Tully et al. 2013; Courtois et al. 2013) example of the cold expansion flows is the local flow of dwarf galaxies (LFD) around the Local Group at the distances of  $\simeq 1-3$  Mpc from the group barycenter. Early observations found the velocity dispersion to be  $\sigma \simeq 60$  km/s in the flow (Sandage et al. 1972; Sandage 1986; Peebles 1988) which is significantly lower than theory expectations (Peebles 1980). The data by Ekholm et al. (1999, 2001) and especially recent high accuracy observations with the Hubble Space Telescope (Karachentsev et al. 2002, 2003a, 2009) give even lower value  $\sigma \simeq 30$  km/s. The other key physical parameter of the flows is the expansion time-rate H; it was found (Sandage et al. 2006; see also references therein) to be equal within 15-20% accuracy to the global cosmological time-rate (Hubble's factor)  $H_0 \simeq 70$  km/s/Mpc. In the LFD, the mean expansion time-rate is 72 km/s/Mpc with 10% accuracy (Karachentsev et al. 2009).

In search for the physical nature of the coldness of the local flows and their nearly cosmological time-rates, two main acting factors have been proposed: the universal dark energy background and the particular local environments of the flows. The studies of the local dark energy effects (Chernin et al. 2000; Baryshev et al. 2001; Chernin 2001) assume that the dark energy is represented by Einstein's cosmological constant  $\Lambda$ , as in the currently standard  $\Lambda$ CDM cosmology. The constant is positive which implies that the dark energy produces a repulsive "force of antigravity". The cosmic repulsion acts not only on the global cosmological distances where it was originally discovered in observations, but actually everywhere in space. In the volume of the Local Group ( $R \leq 1$  Mpc), the dark energy antigravity is weaker than the gravity produced by the matter mass of the group;

but the antigravity repulsion from the group dominates at the distances  $\simeq 1-3$  Mpc outside group's barycenter in the LFD area. The antigravity domination makes the flow expand with acceleration, and it is showed that because of this, the LFD is cooling with time and its time-rate tends to the cosmological value  $H_0$ . These results (Chernin et al. 2000,2004; Baryshev et al. 2001; Karachentsev et al. 2003b; Teerikorpi et al. 2005) are consistent with general astronomical considerations by Sandage et al. (2006) and cosmological simulations by Macció et al. (2005), Peirani & de Freitas (2008), Nurmi et al. (2010). The same physics reveals itself also in several other well studied expansion flows on the spatial scales of 1-10 Mpc (Chernin et al. 2007a,b,c, 2010, Chernin 2013).

On the other hand, Hoffman et al. (2008), Martinez-Vaquero et al. (2009) find the local effect of dark energy to be marginal in constrained N-body simulations of the Local Group. Van de Weigaert & Hoffman (2000) argue that the observed properties of the flow somehow reflect the position of the group in the local universe. While these simulations reproduce to some degree the local cold flow they are not able (Aragon-Calvo et al. 2011) to identify its origin apart from being an intrinsic property of the group environment. The studies of the environment factor take into account the void-wall-filament nature of the large-scale cosmic structure (Klypin et al. 2003; Aragon-Calvo et al. 2011; Einasto et al. 2010, 2012) and suggest that the acceleration and coldness of the local flow may be due to the particular geometry and dynamics of the expanding low-density cosmological wall in which the Local Group resides.

In the present paper, we study the motions of the most luminous nearest galaxies seen at the distances up to 10 Mpc from the barycenter of the Local Group. The galaxies and their numerous fainter companions have recently been observed (Karachentsev et al. 2013, 2014) with the Hubble Space Telescope (HST). Two of the local giants – the Milky Way and the Andromeda Nebula – are moving towards each other inside the Local Group, while a dozen others are moving away

from the group with the radial velocities from 100 to 900 km/s. This is the local flow of giants (LFG), as we refer to it hereafter. Contrary to the LFD, there is no single dominating matter concentration in the LFG and each of the flow members contributes more or less equally to system's total matter mass. We describe the flow motions with the "AN-body" computer model which is a N-body model accounting as well for the universal dark energy background in which the flow is embedded. A combination of the HST data and the model enables us to recognize the dynamical structure of the LFG and the flow major evolutionary trends. Both local effect of dark energy and environment factor acting in the LFG are in the focus of the discussion.

In Sec.2, basic data on the LFG are given in brief; the ΛN-body model is presented in Sec.3; the dynamics and evolution of the flow are analyzed in Secs.4,5, and the results are summarized in Sec.6.

## 2 LFG: Basic data

The observation data on the local giant galaxies and their companions are presented in the recently published Updated Nearby Galaxy Catalogue (UNGC) by Karachentsev et al. (2013) (see also Karachentsev 2005; Karachentsev et al. 2014 and Karachentsev & Kudrya 2014). The catalog contains systematic and homogeneous data on coordinates, distances, radial velocities and other basic physical parameters of about 800 galaxies at the distances up to 10 Mpc. More than 300 UNGC galaxies at these distances have been observed with the HST for more than 300 HST orbits. The unique resolution available in the HST observations allowed to use the Tip of the Red Giant Branch (TRGB) method for precise measurements of distances to more then 300 nearby galaxies with the accuracy of 10%. Precise data on the radial velocities of the galaxies have also been compiled in the UNGC. Modern accurate data on distances and other parameters of nearby galaxies are also accumulated

in the Extragalactic Distance Database (Tully et al. 2008) and in the Database of the Local Volume Galaxies (Kaisina et al. 2012).

According to Karachentsev et al. (2013,2014), there are 15 nearby giant galaxies at the distances up to 10 Mpc: the Milky Way and the Andromeda Nebula (gravitationally bound in the Local Group), galaxies M81, NGC5128, IC342, NGC253, NGC4736, NGC5236, NGC6946, M101, NGC4258, NGC4594, NGC3115, NGC3627, NGC3368 – see Table 1 (MNRAS 449, 2069, 2015).

Each of the giants of Table 1 is actually the main galaxy of a group (similar to the Local Group) which includes the galaxy itself and its extended dark matter halo together with companion galaxies therein. The mass M in the table is the total orbital mass of the group. The UNGC data are used to estimate the masses of the groups via motions of their 351 less massive companions (Karachentsev & Kudrya 2014). The distance R of a giant and its radial velocity V in the table are calculated relative to the barycenter of the Local Group. The Supergalactic coordinates of the giants are also given in Table 1.

The nearest to the Local Group is the M81 galaxy with the distance of 3.6 Mpc. Its recession velocity (relative to the Local Group barycenter) is 100 km/s, the lowest one in the LFG. The mass of the galaxy is  $5 \times 10^{12} M_{\odot}$  (we give here somewhat rounded numbers; the exact figures with the error bars may be found in the paper by Karachentsev & Kudrya, 2014).

The most distant is the galaxy NGC 3368 at 10.4 Mpc with the radial velocity 740 km/s which is the second largest velocity in the LFG. Its mass is  $2 \times 10^{13} M_{\odot}$  which is the second largest mass in the flow. The most massive one is probably the galaxy NGC 4594 at 9.3 Mpc; its mass is  $3 \times 10^{13} M_{\odot}$ , but with the error of  $2 \times 10^{13} M_{\odot}$ , the largest error in the data. The typical mass estimation error for the LFG is not more than 30-40%. The NGC 4594 is the second most distant galaxy with the highest radial velocity of 890 km/s.

As is seen from Table 1, the spatial distribution of 15 LFG giants reveals

a concentration to the Supergalactic plane (see also McCall 2014); the region is referred to as the Local Pan-Cake (Karachentsev et al. 2013). According to Peebles & Nusser (2010) and McCall (2014), 10 giants at distances  $\simeq 1-8$  Mpc form the flattened Local Sheet. The Local Pan-Cake/Sheet looks like part of the large-scale Cosmic Web of walls/sheets/pan-cakes, galaxy clusters, filaments and voids. The Virgo and Fornax clusters at the distances less than 20 Mpc are the closest to the LFG most massive elements of the web. Their masses are  $8 \times 10^{14} M_{\odot}$  and  $1 \times 10^{14} M_{\odot}$ , respectively; the distances are 17 and 20 Mpc, the radial velocities are 975 and 1410 km/s (Karachentsev et al. 2010).

Using the UNGC and the data by Karachentsev & Kudrya (2014), we may make rough quantitative estimates of the gross characteristics of the LFG as a whole system. The mean distance in the sample is  $\langle R \rangle \simeq 7$  Mpc, the mean radial velocity is < V  $>\simeq$  400 km/s, and the mean velocity-to-distance ratio is  $< H > \simeq 65 \; \rm km/s/Mpc$  which is equal – within 10% accuracy – to the cosmological value  $H_0 \simeq 70 \text{ km/s/Mpc}$ . The rms radial velocity dispersion is  $\sigma \simeq 100 \text{ km/s}$ ; the value is considerably less than the mean velocity  $\langle V \rangle$  and the characteristic "virial" velocity of the system  $V_{vir}=[\frac{GM_{tot}}{< R>}]^{1/2}\simeq 300~{\rm km/s}.$  Here  $M_{tot}=8\times 10^{13}M_{\odot}$ is the sum of the matter (dark matter and baryons) masses of the flow member. The mean matter density in the sphere of 10 Mpc in radius  $\langle \rho_M \rangle \simeq 1 \times 10^{-30}$  g cm<sup>-3</sup>, which is about 0.3 the mean cosmological matter density  $\rho_0 \simeq 3 \times 10^{-30} \ \mathrm{g}$ cm<sup>-3</sup>. It indicates (Karachentsev et al. 2012, 2014, Karachentsev & Kudrya 2014) that the volume where the LFG resides is an underdensity region in the local universe. The whole size of the underdensity region reaches the distances of about 50 Mpc from the Local Group (Vennik 1984; Tully 1987; Magtesyan 1988; Bahcall et al. 2000; Crook et al. 2007; Makarov & Karachentsev 2011).

## 3 $\Lambda$ N-body model

With the physical characteristics presented in Sec.2, the LFG looks similar to the LFD or any other local expansion flows: it is rather cold and its mean expansion time-rate is near the global time-rate  $H_0$ . However there are significant differences between the LFG and the LFD. Indeed, the galaxies of the LFD are moving in the external gravity-antigravity field produced by the Local Group and the dark energy, while the mutual gravity of the bodies is practically negligible (Sec.1). The dynamics of the LFG is also controlled by the gravity-antigravity force field, but in contrast to the LFD, there is no single dominating matter concentration in the LFG; each of its member galaxies contributes more or less equally to the total matter mass of the system, and the gravity is due to the mutual attraction of the bodies. The model of the flow we suggest is the  $\Lambda$ N-body model which treats the LFG as a N-body system embedded in the dark energy background.

## 3.1 Local antigravity force

The local dynamical effects of the universal dark energy are actively studied (see for review Chernin 2001, 2008, 2013; Byrd et al. 2008, 2012) since the discovery of the dark energy in cosmological observations. Here we give some basic relations needed for the description of the force field in our model. The relations come from General Relativity with non-zero Einstein's cosmological constant  $\Lambda$  and the currently standard  $\Lambda$ CDM cosmology. The dark energy is considered as a continuous medium with the density  $\rho_{\Lambda} = c^2 \Lambda/(8\pi G) > 0$  that is positive and constant in space and time in any reference frame (here G is the Newtonian gravitational constant and c is the speed of light). The currently adopted value of the dark energy density  $\rho_{\Lambda} \simeq 0.7 \times 10^{-29} \text{ g/cm}^3$ . The dark energy produces the cosmic repulsion, or antigravity, which dominates the dynamics of the observed universe as a whole.

The dark energy as a medium is characterized by the equation of state (Gliner 1965):

$$p_{\Lambda} = -\rho_{\Lambda} c^2. \tag{1}$$

Here  $p_{\Lambda}$  is the dark energy pressure. The dark energy is a vacuum-like medium: it cannot serve as a reference frame, and it is co-moving to any matter motion – similarly to trivial emptiness (Gliner 1965).

In General Relativity, the effective gravitational density is determined by both density and pressure of any fluid:

$$\rho_{eff} = \rho + 3p/c^2. \tag{2}$$

The effective density of dark energy,  $\rho_{\Lambda eff} = \rho_{\Lambda} + 3p_{\Lambda}/c^2 = -2\rho_{\Lambda} < 0$ , is negative, and it is because of this that dark energy produces the repulsion, or antigravity, not gravity.

General Relativity indicates also that the passive gravitational density is the sum  $\rho_{pass} = \rho + p/c^2$  for any fluid. The value is zero for the dark energy. According to the equivalence principle, the passive gravitational mass is equal to the inertial density. Thus, the inertial density of the dark energy is zero. This implies that the dark energy is affected neither by the external gravity of matter nor by its own antigravity.

We assume here that this interpretation of the dark energy may be borrowed from  $\Lambda$ CDM cosmology and applied to relatively small, non-cosmological spatial scales of  $\simeq 1-300$  Mpc, no matter that the isotropic cosmology itself is not valid on these local scales. We will describe the local dynamical effects of dark energy in terms of Newtonian mechanics; it is possible because the velocities of the local flows of galaxies are small compared to the speed of light, and the spatial differences in the gravity-antigravity potential are much smaller (in absolute value) compared to the speed of light squared.

In this weak field approximation, Einstein's law of universal antigravitation says: any body in the universe is affected by the repulsive force which is proportional to the dark energy density  $\rho_{\Lambda}$  and to the distance R of the body from the origin of the adopted reference frame and acts along the direction from the origin:

$$F_{\Lambda} = \frac{8\pi}{3} G \rho_{\Lambda} R. \tag{3}$$

The relation of Eq.3 gives the force per unit mass of the body, i.e. acceleration, in the projection on the body radius-vector. It may be rewritten in the form  $F_{\Lambda} = H_{\Lambda}^2 R$ , where

$$H_{\Lambda} = \left[\frac{8\pi}{3}G\rho_{\Lambda}\right]^{1/2} = 61 \ km/s/Mpc$$
 (4)

is the "universal time-rate" (see below). An exact analytical solution for two-body problem on the dark energy background with the antigravity force of Eq.3 has recently been given by Emelyanov & Kovalyov (2013). The same Eqs.3,4 are used below for the N-body computer solutions.

### 3.2 Equations of motion

In our ΛN-body model, the LFG (together with the two nearest clusters or without them) is treated as a non-relativistic isolated conservative system of point-like masses interacting with each other via Newton's mutual gravity and undergoing Einstein's antigravity (Eq.3) produced by the dark energy background in which the giants and the clusters are embedded. The equations of motion for the system have the following form (see, for comparison, books by Duboshin 1975, and Roy 1978 on N-body problem without the dark energy):

$$\frac{d^2x_i}{dt^2} = G\sum_{i=1}^{N'} m_j \frac{x_j - x_i}{r_{ij}^3} + H_{\Lambda}^2 x_i, \tag{5}$$

$$\frac{d^2y_i}{dt^2} = G\sum_{j=1}^{N'} m_j \frac{y_j - y_i}{r_{ij}^3} + H_{\Lambda}^2 y_i, \tag{6}$$

$$\frac{d^2 z_i}{dt^2} = G \sum_{j=1}^{N'} m_j \frac{z_j - z_i}{r_{ij}^3} + H_{\Lambda}^2 z_i, \tag{7}$$

where

$$r_{ij} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}.$$
 (8)

Here x, y, z are the Cartesian coordinates with the origin in the barycenter of the LFG;  $m_i$  is the mass of the body of the number i, the symbol prime (t) at the summation symbol means the absence of the item with j = i. In Eqs.5-7, Newton's gravity force depends on the distance between gravitating bodies, while Einstein's force depends only on the distance of the body from the coordinate origin.

The number of the LFG bodies is 13, since the Local Group with the Milky Way and the Andromeda Nebula is considered as one mass; another close gravitationally bound binary galaxy, NGC5236 and NGC5128, is also considered as one mass (see Table 1). Together with the two clusters, the total number of the bodies N=15, in the basic model. In the model for the LFD (Sec.5) the total number of bodies N=35.

A numerical integration of Eqs.5-7 is performed with the use of Everhart's (1974) standard computer method with the automatic choice of the integration step.

#### 3.3 Initial conditions

The initial conditions of the motions are specified at the present moment of cosmic time  $t = t_0 = 13.7$  Myr; these are the observed positions of the bodies and their measured radial velocities re-calculated to the barycenter of the Local Group.

Note that observations provide us with the radial velocities of the galaxies, but say nothing about their tangential (transverse) velocities. This is an obvious flaw in any dynamical model of extragalactic astronomy where the full velocity vector is needed for correct formulation of initial conditions. Only for the Andromeda Nebular an estimate of the transversal velocity has recently become available (van der Marel & Guhathakurta 2008). Being cognizant completely about the situation, we assume here for simplicity that the transversal velocities are zero in the initial

conditions for the model (see also Secs. 4,5 below).

### 3.4 Ten integrals

As is well-known (see again Duboshin 1975 and Roy 1978), the N-body equations of motion have 10 first integrals; they correspond to the spacetime symmetries of the equations which imply respective conservation laws. The following quantities must be constant in the N-body motion on the dark energy background:

$$a_x = \sum_{i=1}^{N} m_i x_i, \quad a_y = \sum_{i=1}^{N} m_i y_i, \quad a_z = \sum_{i=1}^{N} m_i z_i,$$
 (9)

$$b_x = \sum_{i=1}^{N} m_i \dot{x}_i, \quad b_y = \sum_{i=1}^{N} m_i \dot{y}_i, \quad b_z = \sum_{i=1}^{N} m_i \dot{z}_i,$$
 (10)

$$c_x = \sum_{i=1}^{N} m_i (y_i \dot{z}_i - z_i \dot{y}_i), \quad c_y = \sum_{i=1}^{N} m_i (z_i \dot{x}_i - x_i \dot{z}_i), \quad c_z = \sum_{i=1}^{N} m_i (x_i \dot{y}_i - y_i \dot{x}_i), \quad (11)$$

$$E = \frac{1}{2} \sum_{i=1}^{N} m_i (\dot{x}_i^2 + \dot{y}_i^2 + \dot{z}_i^2) - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N'} \frac{Gm_i m_j}{r_{ij}} - \frac{1}{2} H_{\Lambda}^2 \sum_{i=1}^{N} m_i (x_i^2 + y_i^2 + z_i^2). \quad (12)$$

The energy conservation law of Eq.12 says that the total mechanical energy of the system is the sum of the total kinetic energy of the bodies and their total potential energy in the gravity-antigravity force field. In the barycentric coordinates used in Eqs.5-8, the values of  $a_x$ ,  $a_y$ ,  $a_z$ ,  $b_x$ ,  $b_y$ ,  $b_z$  must be equal to zero.

The ten integrals of Eqs.9-12 are employed to ensure accuracy in our computer integrations: the integrals are computed and their deviations from the initial values are calculated at the end of each computation session. The integrals prove to be conserved unchanged, and the deviations are found to be zero with all the accuracy of the number presentation in the computer. As for efficiency of integration, the solution of  $\Lambda$ N-body problem with, say, N=15-40 on the cosmic time interval of 15 Gyr takes less than 5 seconds of computing time.

# 4 LFG dynamics and evolution

We have performed computer integration of Eqs.5-7 in two versions. One of them reproduces the system which includes all the giant galaxies of the flow and also two nearest clusters of galaxies (Virgo and Fornax) outside the flow. This is our basic  $\Lambda$ N-body model for the LFG discussed below in this section. In the other version, the system includes the LFG galaxies only. Comparing the models, we may clarify and evaluate the effect of the environment factor in the dynamics of the flow – see below Sec.5. Both models cover the time interval of 15 Gyr, from the cosmic age at t=10 Gyr in its past to the present age of t=13.7 Gyr and further to t=25 Gyr in the future. The history of the flow is traced back in time to narrower spatial configurations of the LFG bodies that still allow their approximate treatment as point-like masses; but deeper in the past, the distances between the bodies would be comparable with their proper sizes. As for the future, the integration is limited by the condition of non-relativistic velocities of expansion. The description of the system for 15 Gyr is seemingly long enough to recognize its dynamical structure and major trends of its evolution.

## 4.1 Phase trajectories

In the basic AN-body model (N=15), the state of the system is given by its position in the 6N-dimension phase space at any instant in the cosmic time interval from 10 to 25 Gyr. For visualization of the results, a two-dimensional V-R hypersurface of system's phase space is used, where  $V \equiv \dot{R}$  is the radial velocity of a body and R is its radial distance – see Fig.1 (in MNRAS 449, 2069, 2015). For this figure, the coordinate transformation is made from system's barycentric (x, y, z) coordinates of Eqs.5-7 to the supergalactic (X, Y, Z) coordinates used in Table.1. Also the radial distances R and radial velocities V are re-calculated to the reference frame of the Local Group barycenter. In Fig.1, the present (observed)

state of the system is showed by the set of black dots. These data are used as "initial conditions" for the integration of the equations of motion. The thick lines in Fig.1 are the phase trajectories of 15 bodies during 15 Gyr of the flow evolution.

It is seen from Fig.1 that the lengths of the phase trajectories are relatively short for the bodies with low initial velocities and long for the high-velocity galaxies and the clusters. All the trajectories demonstrate clearly the major evolutionary trend of the flow: with the distance growth, they converge to the straight line  $V = H_{\Lambda}R$  going from the coordinate origin. As a result, the mean radial velocity dispersion decreases with the distance growth, and the flow gets increasingly regular and cold. Indeed, the spread of the phase trajectories around the line  $V = H_{\Lambda}R$  is systematically decreases with the increase of the distances. For instance, the width of the LFG trajectory bunch decreases by a factor of 3 when the distance R increases from 7.5 to 17 Mpc. This is a clear quantitative measure of the cooling effect in the expansion flow.

The behavior of the phase trajectories in Fig.1 shows also that the flow tends asymptotically to the state in which the trajectories follow the linear velocity-distance relation,  $V = H_{\Lambda}R$ . In this asymptotic state, the flow expansion time-rate  $V/R = H_{\Lambda} = 61 \text{ km/s/Mpc}$  (see Sec.3). This time-rate is constant and depends on the dark energy density only. Its value is not far from the cosmological time-rate  $H_0 \simeq 70 \text{ km/s/Mpc}$  found in global observations (see also below).

On the other distance limit, R < 5 Mpc, relatively small, but obvious deviations from the straight linearity are seen relatively in the phase diagram. These deviations and also intersections of some trajectories there reflect the dynamical effect of the mutual gravitational interactions of the flow bodies. No signs of body binary interactions are seen at the large (R > 10 Mpc) distances in Fig.1. This shows that, with the distance growth, the mutual gravity of the flow bodies becomes weak compared to the repulsion antigravity of the dark energy, and the gravity effect vanishes eventually in the limit of large distances. Thus, the velocity-distance

diagram for the LFG indicates that the dark energy dominates the flow dynamics at these distances.

### 4.2 Asymptotic state

The dynamics of the flow in the limit of large distances where the mutual gravity of the bodies may be neglected is illustrated by an exact analytical solution for non-gravitating particles moving along the radial trajectories in the force field produced by the dark energy alone. The force of the dark energy antigravity is given by Eqs.3,4, and in this limit, the equation of motion for any flow body in the reference frame of the Local Group barycenter has the form:

$$\ddot{R} = \frac{8\pi G}{3} \rho_{\Lambda} R = H_{\Lambda}^2 R. \tag{13}$$

The first integral of the equation

$$\dot{R}^2 = H_{\Lambda}^2 R^2 + 2E,\tag{14}$$

where  $\dot{R}=V$  is the radial velocity, E is a constant which is the total mechanical energy of the particle which may be zero, negative or positive for various trajectories of the flow. When R goes to infinity, Eq.14 gives:

$$V \to H_{\Lambda} R, \quad R \to \infty.$$
 (15)

It is the same asymptotic evolutionary state as is also seen in Fig.1 in the large distance limit: the linear velocity-distance relation with the universal time-rate  $V/R = H_{\Lambda}$ . In this limit, the distances of the flow bodies grow exponentially with time:  $R(t) \propto \exp H_{\Lambda}t$ .

The asymptotic time-rate  $H_{\Lambda}$  of Eq.15 is known for the global cosmological expansion. Indeed, according to the  $\Lambda$ CDM model, the dark energy dominates the global dynamics at the cosmic redshifts  $z < z_{\Lambda} \simeq 0.7$  and the cosmic time  $t > t_{\Lambda} \simeq 7$  Gyr. Since this epoch, the dark energy domination is getting stronger

with time, and in the limit of large times,  $t >> t_{\Lambda}$ , the Friedmann-Lemaître metric tends eventually to the de Sitter metric. In this limit, the Friedmann equation for the cosmological expansion is reduced to the form

$$\ddot{a}(t) = H_{\Lambda}^2 a(t),\tag{16}$$

where a(t) is the cosmological scale factor. This equation is similar to Eqs.13 above, and its solution is similar to Eqs.14,15 for the flow asymptotic state:  $\dot{a}/a = H_{\Lambda}$ ,  $a(t) \propto \exp H_{\Lambda}t$ . Thus, the temporal asymptotic of the global cosmological expansion is exactly the same as the spatial asymptotic of the local expansion flow in the  $\Lambda$ N-body model for the LFG. It is because of this similarity of the global and local that the observed time-rate of the LFG is near the observed value of the Hubble factor, and the both are close to the universal time-rate  $H_{\Lambda}$ .

## 4.3 Spatial trajectories and distances

Alongside with the phase trajectories, the  $\Lambda$ N-body model for the LFG gives the spatial trajectories of the flow giants bodies in the projection to the Supergalactic (Z=0) plane – see thick lines in Fig.2 (MNRAS 449, 2069, 2015). As is seen from the figure, the LFG is expanding monotonically: the distances of the bodies are increasing during all the time of computation, and the mutual distances between the bodies are also increasing. Some of the trajectories are slightly curved at small distances; most of them look like almost straight lines at relatively large distances. This indicates again that the mutual gravity of the bodies is relatively weak in the flow as a whole. In agreement with the results of Fig.1, the spatial trajectories of Fig.2 show that the LFG was rather regular even 2-3 Gyr ago and it is evolving now towards an almost perfectly regular asymptotic state at which mutual binary interactions of the flow members are insignificant.

The  $\Lambda$ N-body model reproduces the distances of the flow bodies from the Local Group barycenter as functions of time from t=10 Gyr to t=25 Gyr – see Fig.3

(MNRAS 449, 2069, 2015). It is seen from the figure that the initially slowest body increases its distance in 1.7 times (from 3.5 to 6 Mpc) during this time; meanwhile the fastest one increases its distance in 3.7 times (from 6 to 22 Mpc). So the difference in radial distances increases in 2.7 times (from 6 to 16 Mpc). This indicates that the geometrical structure of the LFG evolves in a rather complex way at relatively small times. But in the asymptotic regime when the initial conditions are forgot completely, all the distances in the LFG – both radial distances R and mutual distances between the galaxies – increase with time proportionally to the common scale factor  $\exp H_{\Lambda}t$  – like in the cosmological expansion (see above).

It should be reminded here that the tangential (transverse) velocities of the LFG bodies are unknown and we put them to be zero in the initial conditions for the model (Sec.3). Non-zero transverse velocities,  $V_t \neq 0$ , might change the trajectories of the flow in some way; however they could hardly alternate the main trends of the flow evolution and especially the asymptotic state of the flow because the tangential velocities vanish with the growth of the distances, as the conservation of the total angular momentum of the flow (Eq.11) indicates. For each individual body, the product  $V_t R$  is roughly constant in time, and so  $V_t / V \propto R^{-2} \to 0$  when  $V \propto R$  at the limit  $R \to \infty$ . The problem of tangential velocities in the local flows needs, however, more detailed studies.

#### 4.4 Acceleration

Cosmology was sometimes referred to as a science of the two numbers: one of them is Hubble's expansion time-rate  $H(t) = \dot{a}/a$ , and the other is the dimensionless deceleration parameter which is defined as

$$q(t) = -\frac{\ddot{a}a}{\dot{a}^2}. (17)$$

Here a(t) is (as above) the cosmological scale factor. The cosmological deceleration parameter of Eq.17 is positive for redshifts  $z > z_{\Lambda}$  (gravity dominates), it is zero

at  $z = z_{\Lambda}$  (zero-gravity moment) and negative for  $z < z_{\Lambda}$  (antigravity dominates) tending to  $q(t)_{\infty} = -1$  in the limit of infinite cosmological expansion. At the present epoch, the cosmological deceleration parameter  $q_0 \simeq -0.6$ .

The time-rate H(R) was introduced above to the LFG as a local counterpart of Hubble's time-rate in cosmology. In a similar way, we introduce the deceleration parameter q(R) as a function of the radial distance R for the local expansion flows (Chernin & Teerikorpi 2013):

$$q(R) = -\frac{\ddot{R}R}{\dot{R}^2}. (18)$$

In the  $\Lambda$ N-body model of the LFG, the parameter q(R) is calculated for each of the flow trajectories individually. The result is presented in Fig.4 (MNRAS 449, 2069, 2915) where the present-day values of the parameter are showed by black dots. As we see, the deceleration parameter q(R) is negative for each of the flow bodies at all the distances and time moments under consideration. It indicates directly that the LFG is accelerating and the dark energy dominates its dynamics since the cosmic time t=10 Gyr at least. The mean value of the deceleration parameter at present  $< q(R)_0 > \simeq -0.9$ .

The deceleration-distance dependence proves to be, generally, non-monotonic at relatively small distances R < 10 Mpc in Fig.4. It is seemingly due to a well-known sensitivity of few-body or N-body systems to details of initial conditions. Another cause is a complex gravity-antigravity interplay in the flow dynamics. However at larger distances, the initial conditions are mainly forgot, and the mutual gravity vanishes; as a result, all the trajectories tend to the asymptotic value  $q(R)_{\infty} = -1$  when R goes to infinity. This local asymptotic value is the same as the cosmological value  $q(t)_{\infty}$  (see above).

## 5 Local flows: Environment factors

Klypin et al. (2004), Aragon-Calvo et al. (2011), Nurmi et al. (2013) and McCall (2014) argue (see also references therein) that the location of the observed expansion flows is particular and they may be affected by the large-scale Cosmic Web. Here we study two environment factors related to the location of the flows: these are the local underdensity in the large-scale matter distribution and the nearest most massive overdensities in the Cosmic Web which are the Virgo and Fornax clusters of galaxies. We use for these studies the two versions of our computer models for the LFG and also the ΛN-body models for the LFD.

## 5.1 LFG: Underdensity

As is mentioned in Sec.2, the mean present-day matter density  $< \rho_M >_0$  estimated by Karachentsev & Kudrya (2014) for the spherical volume of 10 Mpc in radius around the LFG is about one-third the mean cosmological matter density:  $< \rho_M >_0 \simeq 0.3 \rho_0$  (see Sec.2). The underdensity area extends to distances of 50 Mpc and more being an element of the global Cosmic Web Klypin et al. (2004) and Aragon-Calvo et al. (2011).

In addition to the densities  $\rho_0$  and  $<\rho_M>_0$ , one more characteristic density is also involved in the local flow dynamics – this is the constant in space and time effective gravitating density of the dark energy  $\rho_{eff} = -2\rho_{\Lambda} = -1.4 \times 10^{-29}$  g cm<sup>-3</sup> (see Sec.3). This third density is about 14 times (in absolute value) the mean matter density  $<\rho_M>$  in the LFG area. The ratio  $|\rho_{eff}|/<\rho_M>_0$  may serve as a direct quantitative measure of the dark energy ability to effect the flow. The ratio is high due to the local underdensity at present and it will increase with the LFG expansion in the future. In this way, the local antigravity effect of the dark energy and the environment factor of global underdensity are acting together in same direction enhancing each other: they both makes the flow expand with

acceleration. In the limit of large distances,  $|\rho_{eff}|/<\rho_M>_0\to\infty$  when  $R\to\infty$ .

Is global underdensity needed necessarily for the formation of typical local expansion flows? To discuss this question, we may address the example of the LFD around the Local Group and use it for a comparison with the LFG.

## 5.2 LFD: No underdensity

The LFD is the best studied example of local expansion flows. It demonstrate all the typical features of these system: nearly linear velocity-distance relation, low velocity dispersion, time-rate close to Hubble's global cosmological time-rate (see Sec.1 and references therein). The flow structure, dynamics and evolution are clearly demonstrated by the N-body model for the LFD – see below.

The flow is seen at distances up to 2.7 Mpc from the center of the Local Group. The mean matter density in the spherical volume of 2.7 Mpc in radius is near the cosmological matter density, if not equal to it exactly. Indeed, the mean matter density in this volume is  $M_{LG}/(\frac{4\pi}{3}R_{LFD}^3) \simeq 0.3 \times 10^{-29} \text{ g cm}^{-3}$ , where  $M_{LG} = 3 \times 10^{12} M_{\odot}$  is the matter mass of the group,  $R_{LFD} = 2.7$  Mpc. This coincidence is hardly accidental; rather it suggests that the group and the flow around it occupy the Einstein-Straus vacuole (see Teerikorpi et al. 2008, Saarinen & Teerikorpi 2014 and references therein). Anyway there is no underdensity in the LFD area.

The characteristic ratio  $|\rho_{eff}|/<\rho_M>_0\simeq |\rho_{eff}|/\rho_0\simeq 5$  for the LFD; it is less than that for the LFG, but the dark energy dominates obviously in the LFD area. In the case of the LFD, there is no additional environmental enhancement for the dark energy domination, and nevertheless the flow is expanding with acceleration as is found by Chernin et al. (2000, 2004); see also discussion below and Fig.6 (MNRAS 449, 2069, 2015) where the deceleration parameter q(R) for the LFD is presented which is negative for all the flow bodies. The example of the LFD suggests that a flow may be accelerating even if the mean matter density in its area is not less than the cosmological matter density; but the criterion  $|\rho_{eff}|/<$  $\rho_M>_0\geq 1$  must necessarily be satisfied.

#### 5.3 LFG as environment factor

In our  $\Lambda$ N-body model for the LFD, the flow is surrounded by the local giants together with the two clusters of galaxies. In this way, the LFD environment up to distances of 20 Mpc is included in the consideration. The model is computed with N=20+13+2 for dwarfs, giants and clusters, respectively. The equation of motion of Sec.3 are used together with the corresponding initial conditions given by the observational data (Karachentsev et al. 2009, 2014). The masses of the LFD galaxies are calculated from the data on their red luminosities assuming the typical mass/luminosity ratio m/L=10, in the Solar units. The result is presented by the thick lines in Fig.5 (MNRAS 449, 2069, 2015) where the phase structure of the LFD is given (the LFG and cluster are not showed). The model for the LFD alone (N=20) is also computed – see thin lines in Figs.5,6 (in MNRAS 449, 2069, 2015).

Weak, but clearly visible differences may be found between these two models, especially at relatively small distances and relatively small velocities of the LFD in the diagram of Fig.5. It is seen from Fig.6 that the deceleration parameter is rather sensitive to the environment effect at small distances. However the gross parameters of the accelerating flow as a whole remain generally unchanged. Thus, the LFD structure and evolution does not depend significantly on the presence or absence of the giant galaxies and big clusters around the flow; the dominant role belongs to the dark energy antigravity in the flow.

### 5.4 LFG: Effect of clusters

In our basic  $\Lambda$ N-body model for the LFG with N = 13 + 2 (see Sec.4 above), the two clusters of galaxies – the Virgo and Fornax clusters – are taken into account.

In this way, the gravity of the nearest most massive elements of the large-scale Cosmic Web is directly included into the dynamics of the flow. To visualize the effect of the clusters, we performed computations for a version of the model (N = 13) without the clusters (see the thin lines in Figs.1-4). The comparison with the basic model (the thick lines) shows that in the phase space (Fig.1) the clusters do not affect the trajectories of the bodies with relatively short initial distances. The motions of only two bodies of the LFG with the largest (near 10 Mpc) initial distances are obviously affected as the thin lines indicate this in Fig.1. These two flow trajectories are shifted to relatively higher velocities from the asymptotic trajectory  $V = H_{\Lambda}R$  which enhances somewhat the velocity dispersion in the flow, the two flow trajectories are shifted to relatively higher velocities from the asymptotic trajectory  $V = H_{\Lambda}R$  which enhances somewhat the velocity dispersion in the flow.

In the picture of the spatial trajectories of Fig.2, the effect produced by the clusters is also rather weak, but obvious. Indeed, in the upper part of the diagram, 9 spatial trajectories of the LFG are inclined towards the trajectory of the Fornax cluster compared with their position in the model without clusters. No effect like that is seen from the Virgo cluster. The environmental effect is weak in Fig.3 as well, and it is seen only in the fastest trajectories near the cosmic age of 8 Gyr and more.

The high sensitivity of the deceleration parameter to the initial conditions (as mentioned in Sec.4) is seen in Fig.4. The difference between the models is much larger here than in Figs.1-3. The environmental effect looks especially strong at the distance R < 10 Mpc in the q - R diagram. Interestingly enough, the gravity of the two clusters do not enforce the spread of the curves in this area of the diagram; on the contrary, it introduces rather a specific additional regularity to the flow making the deceleration parameter closer, generally, to the asymptotic value  $q(R)_{\infty} = -1$ . Thus, the general evolution trends of the flow described in

Sec.4 look stable and remain roughly unchanged in the presence or absence of the two closest most massive overdensities of the Cosmic Web.

## 6 Conclusions

The local expansion flow of giant galaxies has recently been discovered and studied with the HST observations at the distances up to 10 Mpc (Karachentsev et al. 2013, 2014, Karachentsev & Kudrya 2014). The flow involves dozen most luminous nearby galaxies together with about 300 their fainter companions. The flow occupies a flattened volume near the Supergalactic Plane. The flow is expanding: the giants are moving away from the barycenter of the Local Group of galaxies with the radial velocities from 100 to about 1000 km/s. The total matter mass of the flow is  $M \simeq 8 \times 10^{13} M_{\odot}$ .

We have developed a set of computer models which treat the local flow of giant as a N-body system embedded in the perfectly uniform time-independent background of the dark energy represented by Einstein's cosmological constant  $\Lambda$ . In these " $\Lambda$ N-body"models, the dynamical state of the flow and the trends of its evolution are controlled by the local interplay between the mutual gravity of the galaxies and the cosmic antigravity produced by the dark energy. A combination of the observational data with the computer models leads to following conclusions:

- 1) The local flow of giants (LFG) reveals two major observed features which are common according to Sandage (1986), Sandage et al. (1972, 1999, 2006) for expansion flows of various spatial scales: the nearly linear velocity-distance relation  $V \simeq HR$  and the nearly cosmological mean value of the expansion timerate  $\langle H \rangle \simeq 65 \text{ km/s/Mpc}$ .
- 2) The third major feature of the LFG is its acceleration which is quantified by the dimensionless deceleration parameter,  $q(R) = \ddot{R}/(VR)$  borrowed from cosmology. The parameter proves to be negative for each of the LFG individual

galaxies since the cosmic time t=10 Gyr at least. Its present mean value for the flow as a whole is  $< q(R) > \simeq -0.9$ .

- 3) The negative deceleration parameter means that the flow is accelerating, and the flow acceleration indicates in its turn that the flow dynamics is dominated by the antigravity produced by the dark energy.
- 4) A direct dimensionless quantitative measure of the dark energy domination in the LFG is the ratio of the effective gravitating density (in absolute value) to the mean matter density in the flow area  $|\rho_{eff}|/<\rho_M>$ . The present-day value of the ratio is  $\simeq 14$ , which is about three times larger then the similar ratio found earlier for the local flow of dwarf galaxies around the Local Group at the distances less than 3 Mpc. In the LFG, the dynamical effect of the dark energy is enhanced by the environment factor of the low mean matter density existing on the large scale of 50 Mpc in the local universe.
- 5) In the limit of large distances when  $R \to \infty$ , the flow time-rate tends to the value  $H_{\Lambda} = [\frac{8\pi}{3}G\rho_{\Lambda}]^{1/2} = 61 \text{ km/s/Mpc}$  which depends on the dark energy density only. Alongside with this, the deceleration parameter of each individual galaxies of the flow goes to the common value q(R) = -1. The density ratio  $|\rho_{eff}|/<\rho_M>$  tends to infinity in this limit. The values  $H_{\Lambda}$ ,  $q_{\Lambda} = -1$  and the infinite density ratio  $|\rho_{eff}|/\rho_M$  are also known as asymptotic characteristics of the global cosmological expansion in the limit when  $t \to \infty$ . In terms of General Relativity, both local and global expansion flows have the same asymptotic 4-dimensional state which is the de Sitter space-time.
- 6) The environment conditions related to the large-scale Cosmic Web with its most massive nearby matter concentrations (the Virgo and Fornax clusters of galaxies at distances about 20 Mpc) are able to modify slightly details of the LFG internal structure.

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# Figure captions

- Fig.1. Local flow of giants: Velocity-distance diagram. The thin lines are for the model without the clusters of galaxies. Solid line is the asymptotic linear relation  $V = H_{\Lambda}R$ . Black dots are for the present-day values.
- Fig.2. Local flow of giants: The spatial trajectories of the LFG galaxies in the projection to the Supergalactic (Z=0) plane. The thin lines are for the model without the clusters. Black dots are for the present-day values.
- Fig.3. Local flow of giants: The distances of the LFG galaxies from the Local Group barycenter. The thin lines are for the model without the clusters.
- Fig.4. Local flow of giants: The deceleration parameter as a function of the radial distance of the LFG galaxies. The thin lines are for the model without the clusters. Black dots are for the present-day values.
- Fig.5. Local flow of dwarfs: Velocity-distance diagram. The thin lines are for the model without the LFG and clusters. Solid line is the asymptotic linear relation  $V = H_{\Lambda}R$ . Black dots are for the present-day values.
- Fig.6. Local flow of dwarfs: The deceleration parameter as a function of the radial distance of the LFD galaxies. The thin lines are for the model without the LFG and the clusters. Black dots are for the present-day values.